## Time Value of Money Calculation Methods

The time value of money may be calculated using a variety of techniques. When achieving specific financial goals requires regular deposits to a savings or investment account, the computation may occur in one of several ways. For example, Jonie plans to deposit $10,000 in an account for the next 10 years. She estimates these funds will earn an annual rate of 5 percent. What amount can Jonie expect to have available after 10 years?

$$PV = -100000$$

$$FV = 16,288.95$$

$$I/y = 5$$

$$N = 10 \quad F$$

$$FV = PV(1+r)^t$$

$$FV = 10000(1+.05)^{10}$$

$$FV = 16,288.95$$

# The Time Value of Money: Future Value and Present Value Computations

"If I deposit $10,000 today, how much will I have for a down payment on a house in five years?"

"Will $2,000 saved a year give me enough money when I retire?"

"How much must I save today to have enough for my children's post-secondary education?"

As introduced in **Chapter 1** and used to measure financial opportunity costs in other chapters, the *time value of money*, more commonly referred to as *interest*, is the cost of money that is borrowed or lent. Interest can be compared to rent, the cost of using an apartment or other item. The time value of money is based on the fact that a dollar received today is worth more than a dollar that will be received one year from today because the dollar received today can be saved or invested and will be worth more than a dollar a year from today. Similarly, a dollar that will be received one year from today is currently worth less than a dollar today.

The time value of money has two major components: future value and present value. *Future value* computations, which are also referred to as *compounding*, yield the amount to which a current sum will increase based on a certain interest rate and period of time. *Present value*, which is calculated through a process called *discounting*, is the current value of a future sum based on a certain interest rate and period of time.

In future value problems, you are given an amount to save or invest and you calculate the amount that will be available at some future date. With present value problems, you are given the amount that will be available at some future date and you calculate the current value of that amount. Both future value and present value computations are based on basic interest rate calculations.

1000                        4000

500 PV
+ 50/mo

PV    PMT

# FINANCIAL CALCULATORS

Currently, financial calculators, with time value of money functions built in, are widely used to calculate future value, present values, and annuities. For the following examples, we will use the Texas Instruments BA II Plus financial calculator, which is recommended by the Canadian Institute of Financial Planning.

When using the BA II Plus calculator to solve time value of money problems, you will be working with the TVM keys that include:

*2nd M-CLR (2nd + Alpha) to clear lingring data*

[CPT] – Compute key used to initiate financial calculations once all values are input

[N] – Number of periods

[I/Y] – Interest rate per period

[PV] – Present value

[PMT] – Amount of payment, used only for annuities

[FV] – Future value

Enter values for PV, PMT, and FV as negative if they represent cash outflows (e.g., investing a sum of money) or as positive if they represent cash inflows (e.g., receiving the proceeds of an investment). To convert a positive number to a negative number, enter the number and then press the +/– key.

The examples that are shown in this chapter assume that interest is compounded annually and that there is only one cash flow per period. To reflect this, we must set the number of payments and compounding per period to 1 (the default setting is 12). To do this, press in turn the [2nd] button (yellow), the [N] button (for the P/Y function shown above it), the number 1, the [ENTER] button, the [2nd] button again, and finally the [CPT] button (for the quit function above it). Before using any financial calculator, we strongly recommend that you consult the instruction manual that accompanies it and attempt the examples shown there.

Now let's try a problem. What is the future value of $100 after three years at a 10 percent annual interest rate? Remember that an investment of money is considered to be an outflow of cash; therefore, the present value of $100 should be entered as a negative number.

First, you must enter the data. Remember that an investment of money is considered to be an outflow of cash; therefore, the $100 should be entered as a negative number.

| | | |
|---|---|---|
| 3 | [N] | |
| 10 | [I/Y] | |
| 100 +/– | [PV] | |
| 0 | [PMT] | (optional if registers are cleared) |

*PV = -100*
*N = 3*
*I/Y = 10*
*P/Y = 1*
*FV = 133.10*

To find the solution, the future value, press [CPT] [FV], and the future value of 133.1 is displayed.

$$FV = PV(1+r)^t$$
$$100(1+.10)^3$$

# FUTURE VALUE OF A SINGLE AMOUNT

The future value of an amount consists of the original amount plus compound interest. This calculation involves the following elements:

$FV$ = Future value
$PV$ = Present value
$i$ = Interest rate
$n$ = Number of time periods

*handwritten:* P/Y

*handwritten:* PV=400   N=18
I/y=12/12 = 1
FV=$478.46

**Example B:** If your savings of $400 earns 12 percent, compounded monthly, over a year and a half, use the table factor for 1 percent for 18 time periods; the future value is:

$478.46 = $400(1 + 0.01)^{18}$   $478.40 = $400(1.196)$   400 [PV], 12/12 = 1 [I/Y], 1.5 × 12 = 18 [N], 0 [PMT] [CPT] [FV] 478.46

**Sample Problem 1** What is the future value of $800 at 8 percent after six years?

*handwritten:* PV= -800   N=6   1269.50 ✓
I/y = 8   FV=

**Sample Problem 2** How much would you have in savings if you kept $200 on deposit for eight years at 8 percent, compounded *semi-annually?*

*handwritten:* PV= -200   FV= 374.60 ✓
I/y = 8/2 = 4
N = 8×2 = 16

# FUTURE VALUE OF A SERIES OF EQUAL AMOUNTS (AN ANNUITY)

Future value may also be calculated for a situation in which regular additions are made to savings. The formula and financial calculator computations are as follows:

*handwritten:* PMT = -40   N=10   I/y =8   FV= $579.46

**Example D:** If you plan to deposit $40 a year for 10 years, earning 8 percent compounded annually, the future value of this amount is:

$$579.46 = \frac{$40(1 + 0.08)^{10} - 1}{0.08}$$   Using Exhibit 1B-2 @ :
$579.48 = $40(14.487)$   -40 [PMT], 10 [N], 10 [I/Y], 0 [PV] [CPT] [FV] 579.46

**Sample Problem 3** What is the future value of an annual deposit of $230 earning 6 percent for 15 years?

*handwritten:* PMT = 230   I/y = 6   N=15   FV= 5353.47 ✓

**Sample Problem 4** What amount would you have in a retirement account if you made annual deposits of $375 for 25 years earning 12 percent, compounded annually?

*handwritten:* PMT= -375
N= 25
I/y = 12
FV= $50 000.20

*Calculate PV*

# PRESENT VALUE OF A SINGLE AMOUNT

If you want to know how much you need to deposit now to receive a certain amount in the future, the formula and financial calculator computations are as follows:

$FV = 300$    $I/Y = 10/2 = 5$    $N = 7 \times 2$

**Example F:** If you want to have $300 seven years from now and your savings earn 10 percent, compounded semi-annually (5 percent for 14 time periods), finding how much you would have to deposit today is calculated as follows:

$$\$151.52 = \frac{\$300}{(1+0.05)^{14}}$$    Using Exhibit 1B-3 ⓐ:   $\$151.50 = \$300(0.505)$    300 FV, 7×2 = 14 N, 10/2 = 5 I/Y, 0 PMT, CPT, PV −151.52

$PV = -151.52$ ✓

**Sample Problem 5** What is the present value of $2,200 earning 15 percent for eight years?

$PV = $ _____    $FV = 2200$    $I/Y = 15$    $N = 8$    (719.18 ✓)

**Sample Problem 6** To have $6,000 for a child's education in 10 years, what amount should a parent deposit in a savings account that earns 12 percent, compounded *quarterly*?

$I = 12/4 = 3$    6000

$N = 10yrs \times 4$
$40$

$PV = 1839.34$

# PRESENT VALUE OF A SERIES OF EQUAL AMOUNTS (AN ANNUITY)

The final time value of money situation allows you to receive an amount at the end of each time period for a certain number of periods. The formula and financial calculator computations are as follows:

**Example H:** If you wish to withdraw $100 at the end of each year for 10 years from an account that earns 14 percent, compounded annually, what amount must you deposit now?

$PV = 521.61$

$$\$521.61 = \$100 \left[ \frac{1 - \frac{1}{(1+0.14)^{10}}}{0.14} \right]$$   Using Exhibit 1B-4 ⓐ:   $\$521.60 = \$100(5.216)$   100 PMT, 10 N, 14 I/Y, 0 PV, CPT, PV (521.61156)

$\$100 = PMT$

$PV$    $N = 10yrs$    $I/Y = 14$

**Sample Problem 7** What is the present value of a withdrawal of $200 at the end of each year for 14 years with an interest rate of 7 percent?

$PV = $ _____    $PMT = 200$    $N = 14$    $I = 7$    $PV = 1749.09$

**Sample Problem 8** How much would you have to deposit now to be able to withdraw $650 at the end of each year for 20 years from an account that earns 11 percent?

$PV = 5176.16$

$PMT = 650$
$N = 20$
$I/Y = 11$

# USING PRESENT VALUE TO DETERMINE LOAN PAYMENTS

| Present Value to Determine Loan Payments | |
|---|---|
| Table | Financial Calculator |
| | [N], [I/Y], [PV], [FV], [CPT], [PMT] |
| Example I: If you borrow $1,000 with a 6 percent interest rate to be repaid in three equal payments at the end of the next three years, the payments will be $374.11. This is calculated as follows: $$\frac{\$1,00}{2.673} = \$374.11$$ | 1000 [PV], 6 [I/Y], 3 [N], 0 [FV], [CPT] [PMT] 374.10981 |

*(handwritten, right margin)*
$PV=1000$
$I/Y=6$
$N=8$
$PMT = \underline{374.11}$

**Sample Problem 9** What would be the annual payment amount for a $20,000, 10-year loan at 7 percent?

*(handwritten)*
$PMT = \underline{\qquad}$
$PV = 20000$
$N = 10$
$I/y = 7$

$\$2847.55$

## Financial Planning Calculations

### Annual Contributions to Achieve a Financial Goal

Achieving specific financial goals often requires regular deposits to a savings or investment account. By using time value of money calculations, you can determine the amount you should save or invest to achieve a specific goal for the future.

### EXAMPLE 1

Jonie has two children who will start post-secondary education in 10 years. She plans to set aside $1,500 a year for her children's education during that period and estimates she will earn an annual interest rate of 5 percent on her savings. What amount can Jonie expect to have available for her children's post-secondary education when they are ready to enroll?

*(handwritten)*
$N = 10$  $PMT = 1500/yr$  $I/y = 5$  $FV = \underline{\qquad}$
$\$18866.84$

### EXAMPLE 2

Don wants to accumulate $50,000 over the next 10 years as a reserve fund for his parents' retirement living expenses and health care. If he earns an average of 8 percent on his investments, what amount must he invest each year to achieve this goal?

# PRESENT VALUE OF A SERIES OF DEPOSITS

You can also use present value computations to determine how much you need to deposit so that you can withdraw a certain amount from the account for a desired number of years. For example, if you want to take $400 out of an investment account each year for nine years and your money is earning an annual rate of 8 percent, you can see from Exhibit 1–8D ⊘ that you need to make a current deposit of $█████████

$$PMT = 400/YR \quad I/ = 8$$
$$N = 9 \quad PV = \boxed{2498.76}$$

## CONCEPT CHECK 1–4

1. How can you use future value and present value computations to measure the opportunity cost of a financial decision? *Give up Money for future*

2. Use the time value of money tables in Exhibit 1–8 ⊘ or a financial calculator to calculate the following:
   a. The future value of $100 at 7 percent in 10 years. $FV = 100(1+.07)^{10} = 196.72$
   b. The future value of $100 a year for six years earning 6 percent. $100(1+.06)^6 = 141.85$
   c. The present value of $500 received in eight years with an interest rate of 8 percent.

   $PV =$ ___ $FV = 500 \quad N = 8 \quad I = 8$

   $\boxed{\$270.13}$

   $\# 91461.53$

## PRACTICE PROBLEMS

1. *Calculating Future Value of Property.* Ben Collins plans to buy a house for $65,000. If that real estate property is expected to increase in value by 5 percent each year, what will its approximate value be seven years from now? LO3

2. *Using the Rule of 72.* Using the rule of 72, approximate the following amounts: LO3 $72/6$ PV
   a. If land in an area is increasing 6% a year, how long will it take for property values to double? $10000$ PV $20000$
   b. If you earn (10) percent on your investments, how long will it take for your money to double? $72/10$
   c. At an annual interest rate of 5 percent, how long will it take for your savings to double? $72/6$

3. *Determining the Average Price Increase.* A car that cost $12,000 in 1998 cost $16,000 10 years later. What was the rate of increase in the cost of the car over the 10-year period? LO3

$$PV = 12000$$
$$FV = 16000$$
$$N = 10$$

$\boxed{2.9\%}$ /10yrs

4. *Determining the Required Deposit.* If you want to have $7,000 in five years, how much do you have to deposit today if your investment earns a rate of 3 percent per annum? LO3

5. *Determining Interest Rates.* The Benevolent Company has agreed to lend you funds to complete the last year of your degree. The Company will lend you $2,400 today if you agree to repay a lump sum of $4,000 four years from now. What annual rate of interest is the Company charging you? LO3

6. *Calculating Future Value.* How long will it take to double your money with a growth rate of 5 percent and 12 percent respectively? LO3

7. *Determining the Number of Years.* You discover $40,000 under your pillow, which can be invested at a rate of 18 percent per year. If you spend $11,435 per year, how long will the money last? LO3

*Handwritten work:*

$n = 6$

**4.** $FV = 7000$
$N = 5$
$I/Y = 3$
$PV =$ **6038.26**

**5** $PV = 2400$
$FV = 4000$
$N = 4$
$I/Y = $ **14%**

**6**
$\frac{72}{5} = $ **14.4**
$\frac{72}{12} = $ **6**

**7.**
$PV = 40000$
$PMT = 11435$
$I/Y = 18$
$N = 6$

8. *Calculating Annual Payments.* What annual payment is required to pay off a four-year, $20,000 loan if the interest rate being charged is 7 percent? LO3

9. *Determining the Income Flow.* You have $100,000 to invest today. At 5 percent per year, what sum can you withdraw at the end of each year, for a period of 20 years, before your money is exhausted? LO4

10. *Exploring Other Time Value of Money Applications.* Using time value of money tables or a financial calculator, calculate the following: LO4

  a. The future value of $450 six years from now at 7 percent.

  b. The future value of $800 saved each year for 10 years at 8 percent.

  c. The amount you have to deposit today (present value) at a 6 percent interest rate to have $1,000 five years from now.

  d. The amount you have to deposit today to be able to take out $500 a year for 10 years from an account earning 8 percent.

11. *Calculating Future Value of a Series of Amounts.* Elaine Romberg prepares her own income tax return each year. A tax preparer charges her $60 for this service. Over a period of 10 years, how much does Elaine save from preparing her own tax-return? Assume she can earn 6 percent with a savings certificate. LO4  $PMT = 60 \quad N = 10 \quad I/Y = 6 = $ **790.86**

*Handwritten work:*

**8** **5904.56**
$PMT = $
$N = 4$
$PV = 20000$
$I/Y = 7$

**9** $PV = 100\,000$
$I/Y = 5$
$PMT = $
$N = 20$
**8024.26**

**10**
**9** $FV = 675.45$
$PV = 450$
$N = 6$
$I/Y = 7$

**b** $FV = 1589.60$
$PMT = 800$
$N = 10$
$I/Y = 8$

**c** $PV = 747$
$I/Y = 6$
$FV = 1000$
$N = 5$

**d** $PV = 3355$
$PMT = 500$
$N = 10$
$I/Y = 8$

12. *Calculating the Future Value of a Single Sum.* You have $800 in a savings account that earns 6% interest compounded annually. How much additional interest would you earn in two years if you moved the $800 to an account that earns 6 percent compounded semi-annually? LO4

13. *Calculating the Future Value of a Single Sum.* What is the future value of $20,000 received in 10 years if it is invested at 6 percent compounded annually for the next six years and at 5 percent compounded annually for the remaining four years? LO4

14. *Calculating the Present Value of a Single Sum.* Your parents have promised to give you a graduation present of $5,000 when you graduate in four years. If interest rates stay at 6 percent compounded annually for the next four years, how much is this money worth in today's dollars? LO4

12   $PV = 800$   $898.88$      $8000$    $900.41$
     $I/Y = 6$                  $I/ = \frac{6}{2} 3$      → DIFF
     $N = 2$                    $N = 4 (2 \times 2$        $1.53$

13   $20000 \longrightarrow 6\% I/Y$
     $6N$
     $28370.38 \longrightarrow$   $5\% IY \longrightarrow 34484.37$
                                   $4N$

14   ⟨453⟩            $548.0$
     $10\%$
     $PV$            $12\%$          $FV = 687.42$
                     $PV$

**MULTIPLE CHOICE.** **Choose the one alternative that best completes the statement or answers the question.**

1) Assume your uncle will pay you $100 for each of the next two years and $200 in years 3 and these amounts will be paid at year end. Assume the interest rate is 10% for the first two years and 12% for the next two (years 3 and 4). What is your uncle's promise worth in today's dollars? (Round your answer)    1) _____

A) $600      B) $317      C) $512      D) $453      E) $342      $200 PMT = N = 2 I/Y = R = 0 \boxed{ . }$

$PMT = 100 \quad N = 2 \quad I/Y = 10 \quad = FV = \boxed{\quad} \longrightarrow 12\% 2yrs \boxed{ . } FV$

2) What is the future value of $20,000 received today, after 10 years if it is invested at 6% compounded annually for the next six years and 5%, compounded semi-annually for the remaining four years? 2) _____

A) $32,772    B) $38,817    C) $25,000    D) $34,567    E) $31,000

$PV 20000 \quad I/Y = 6 \qquad PV = \rightarrow \underline{\qquad} \qquad FV = \underline{\qquad}$
$N = 106 \quad FV = \underline{\qquad} \qquad I/Y 5/2 \quad N = 4 \times 2$

3) What is the future value of $30,000 received today, after 10 years if it is invested at 7% compounded annually for the next seven years and 5%, compounded annually for the remaining three years?   3) _____

A) $71,000    B) $54,567    C) $81,744    D) $62,772    E) $55,767

$PV = 30000$              $PV = 48173.44$
$N = 107 \quad 4817344$    $N = 3$
$I/Y = 7$                  $I/Y = 5 \quad 55766.7$
$\quad FV = $              $FV = $

4) What is the future value of $80,000 received today, after 14 years if it is invested at 8% compounded annually for the next five years and 3%, compounded annually for the remaining nine years?     4) _____

A) $171,022
B) $158,098
C) $153,371
D) $134,567
E) $144,772

D

$PV = 80000$
$N = 5$
$I/y = 8$
$FV = $ _____

$PV = $ _____
$I/y = 3$
$N = 9.$
$FV = $ _____

5) If a person deposited $10,000 earning 9 percent for 11 years, this would involve what type of computation?     5) _____

A) present value of a single amount
B) simple interest
C) future value of a series of deposits
D) present value of a series of deposits
E) future value of a single amount

E

$PV = 1000$

6) An individual invests $10,000 at a rate of 5% per annum. What will be its value in 10 years' time?     6) _____

A) $15,853     B) $15,000     C) $19,000     D) $18,000     E) $16,289

E

$PV = 10000$
$I = 5$
$N = 10$

7) Your goal is to accumulate in 4 years $5,000. If you can earn a rate of 4%, compounded monthly, what will be your end of month monthly payment need to be to reach this goal?     7) _____

A) $124     B) $300     C) $104     D) $96 E) $262

D

$N = 4 \times 12 = 48$
$I/y = 4/12 = $
$FV = 5000$
$PMT = $ _____

8) Your goal is to pay down your student loan in 3 years. The balance today is $9,434. If you are charged a rate of 4%, compounded monthly, what will be your monthly, end-of-period payment? 8) _____

A) $279     B) $406     C) $300     D) $262     E) $377

A

$N = 12 \times 3$
$I/y = 4/12$
$PV = 9434$
$PMT = $ _____

9) An individual invests $5,000 at a rate of 5% per annum. What will be its value in 10 years' time?     9) _____

A) $9,000     B) $8,144     C) $9,542     D) $7,500     E) $7,927

$PV = 5000 \quad I/y = 5 \quad N = 10$     B

10) Assume your friend will pay you $200 for each of the next two years and $400 in years 3 and these amounts will be paid at year end. Assume the interest rate is 10% for the first two years and 12% for the next two (years 3 and 4). What is your friend's promise worth in today's dollars? (Round your answer)     10) _____

A) $951     B) $831     C) $1,000     D) $906     E) $600

$N = 2$
$PMT = 200$
$I/y = 10$
$FV = \_\_\_$     $N = 2$  $+ 12\% = PV = \_\_\_$

D

11) Your goal is to pay down your student loan in 3 years. The balance today is $9,434. If you are charged a rate of 9%, compounded monthly, what will be your monthly, end-of-period payment?     11) _____

A) $527     B) $406     C) $193     D) $300     E) $262

$PMT = \_\_\_ \quad I/y = 9$
$PV = 9434 \quad N = 3 \times 12$     D

12) You wish to accumulate $15,000 within five years. How much would you have to save each year for five years to attain your goal? Assume an annual interest rate of 4%. Savings occur at the end of each year.     12) _____

A) $3,500     B) $2,662     C) $3,000     D) $2,905     E) $2,769

$FV = 15000 \quad I/y = 4$
$N = 5 \quad PMT = \_\_\_$     E

13) An individual invests $12,000 at a rate of 4% per annum. What will be its value in 9 years' time?     13) _____

A) $15,853     B) $15,000     C) $18,000     D) $17,080     E) $16,289

$PV = 12000 \quad N = 9$
$I/y = 4 \quad FV = \_\_\_$     D

14) An individual invests $9,000 at a rate of 6% per annum. What will be its value in 11 years' time?     14) _____

A) $15,000     B) $17,085     C) $18,000     D) $16,289     E) $15,853

$PV = 9000 \quad N = 11$
$I/y = 6 \quad FV = \_\_\_$     B

15) If a person deposited $100 a month for 5 years earning 9 percent, this would involve what type of computation?  15) _____

$PMT = 100$

A) present value of a single amount
B) present value of a series of deposits
C) future value of a single amount
D) simple interest
E) future value of a series of deposits

E

$400/mo \quad \times 34 yrs$
$\dfrac{26 - 60}{34} \quad \dfrac{163200}{500000}$